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Research Article

Consistency of the Base-Ten System in Counting-Grid Visuals: A Content Analysis of First-Grade Elementary School Mathematics Textbooks

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ABSTRACT

This study aims to analyze the consistency of the number bases used in counting-grid visual media in Grade 1 elementary school mathematics textbooks published by the Indonesian Ministry of Education, Culture, Research, and Technology, and to formulate instructional media solutions that ensure such consistency. The analysis covered two textbooks, namely Tim Gakko Kasho (2021) and Wulan (2023). The study employed content analysis using a qualitative-descriptive approach. The findings indicate that Tim Gakko Kasho (2021) uses counting-grid media with inconsistent number bases, including bases 5, 7, 9, and 10. Some pages even present two different bases simultaneously. Meanwhile, Wulan (2023) uses a 2 × 5 grid format that structurally develops base 5 before base 10. Such inconsistencies in iconic representation may lead to misconceptions about number bases, conflicts in cognitive schemas, and long-term difficulties in mastering place value and arithmetic operations in subsequent grades. Drawing on the Concrete–Representational–Abstract (CRA) framework, which aligns with Bruner's theory of representation, and supported by the theories of Van de Walle, Ma Liping, Clements and Sarama, Piaget, and Kilpatrick et al., this article recommends the use of a 1 × 10 counting-table card with buttons as concrete or enactive media, and a 1 × 10 counting-grid image completed with writing tools as semi-concrete or iconic media. These media are proposed as solutions to ensure consistent use of the base-ten system across all stages of representation.

Keywords: Base Ten; Counting; Counting-Grid Visual Media; Counting-Table Cards; Concrete Media; Concrete–Representational–Abstract (CRA); Textbook Analysis; Misconceptions; Grade 1 Elementary School

1. INTRODUCTION

Counting is one of the earliest mathematical competencies that first-grade elementary school students must master. This competency involves more than merely reciting a sequence of numbers. It requires students to develop conceptual understanding that serves as the foundation for the entire hierarchy of subsequent mathematical knowledge, including numbers; number comparisons involving equal to, greater than, less than, and zero; basic arithmetic operations such as addition, subtraction, multiplication, and division; advanced arithmetic operations such as exponentiation and root extraction; place value and place-value-based operations; integers and their operations; elementary algebra involving numerical patterns and sequences; and fractions and their operations. A strong foundation at this stage determines students' long-term success in mathematics (Kilpatrick, Swafford, & Findell, 2001).

Visual instructional media play a strategic role in learning to count. Bruner (1966) asserts that children learn through three modes of representation: enactive representation through real objects, iconic representation through images, and symbolic representation through numbers. Iconic representation, particularly counting-grid visuals, provides a critical bridge between concrete experiences and abstract understanding. Therefore, the visual structure of iconic media used during the initial stage of learning must remain consistent and coherent with the mathematical concepts being developed.

The number system used universally is the decimal system, which is based on ten. All elementary school content concerning numbers and arithmetic operations, from ones, tens, and hundreds place values to arithmetic calculations, relies on the base-ten system. Therefore, counting media introduced in the first grade should consistently develop a base-ten cognitive schema in students' minds. However, an examination of first-grade mathematics textbooks published by the Indonesian Ministry of Education, Culture, Research, and Technology reveals a different situation. A content analysis of two major textbooks, namely Tim Gakko Kasho (2021) and Wulan (2023), identified inconsistencies in the number bases represented through counting-grid visuals. Gakko Kasho (2021) alternately uses counting grids based on 5, 7, 9, and 10.

Some pages even present two different bases simultaneously. Meanwhile, Wulan (2023) consistently uses a 2×5 grid format, which indirectly develops base-5 grouping before establishing base 10. According to previous scholars, such inconsistencies in iconic representation may generate several negative consequences, including misconceptions about number bases, conflicts between cognitive schemas, and long-term difficulties in mastering place value and arithmetic operations in subsequent grades (Ashlock, 2010; Clements & Sarama, 2004; Ma Liping, 1999).

Based on this background, this article aims to: (1) describe the inconsistencies in the number bases used in counting-grid visuals in two first-grade elementary school mathematics textbooks published by the Indonesian Ministry of Education, Culture, Research, and Technology; (2) analyze the pedagogical and cognitive effects of these inconsistencies using relevant theoretical foundations; and (3) formulate concrete and semi-concrete instructional media solutions that ensure base-ten consistency across all stages of representation as alternatives and supplements to the existing textbooks.

2. THEORETICAL REVIEW

2.1 Counting and the Construction of Base-Ten Understanding

Counting is not merely an activity that involves memorizing number sequences. Gelman and Gallistel (1978) identified five counting principles that children must master simultaneously: the one-to-one correspondence principle, the stable-order principle, the cardinality principle, the abstraction principle, and the order-irrelevance principle. These principles demonstrate that counting is a complex cognitive activity rather than a simple process of naming numbers. Within the framework of the Kurikulum Merdeka, counting competencies for first-grade elementary school students in Phase A focus on developing an understanding of numbers from 1 to 10, which subsequently expands to numbers up to 20 and 100. The primary purpose is to prepare students to understand place value, which relies entirely on the decimal or base-ten system. Therefore, every visual representation used in counting instruction should consistently reflect the structure of base ten. Van de Walle, Karp, and Bay-Williams (2013) recommend the ten-frame as a standard instructional medium for early counting. A ten-frame consists of ten boxes that help students visualize the relationships between numbers and ten, understand number composition and decomposition, and develop basic addition facts. However, Van de Walle also distinguishes between two ten-frame formats: the 1×10 format, which consists of one row of ten boxes and directly develops base-ten understanding, and the 2×5 format, which consists of two rows of five boxes and initially constructs base-5 grouping before base-ten grouping.

2.2 Assimilation, Accommodation, and Schema Stability

Piaget (1965) explains that children construct number concepts through two primary cognitive mechanisms: assimilation, which involves integrating new information into an existing schema, and accommodation, which involves modifying an existing schema to incorporate information that does not fit. Healthy schema development occurs when new information remains compatible with previous experiences, allowing assimilation to proceed smoothly. When counting media use different number bases, such as 5, 7, 9, and 10, students must repeatedly engage in unguided accommodation. Each change in the number base forces the child's mind to modify the numerical schema that is still under construction. Consequently, the resulting schemas become unstable and incoherent. Instead of developing one strong base-ten schema, children may construct several partial schemas that compete with one another.

2.3 Consistency Across Enactive, Iconic, and Symbolic Representations

Bruner (1966) emphasizes that the three modes of representation, namely enactive, iconic, and symbolic representation, must share the same underlying structural connection. Iconic representations, such as counting-grid visuals, must reflect the same mathematical structure that students are expected to develop at the symbolic level, particularly the decimal number system. When iconic representations use changing structures based on 5, 7, 9, and 10, no consistent structural connection can link students' visual experiences to their symbolic understanding of the decimal system. Ding and Li (2014) state that this inconsistency may hinder the transition from iconic to symbolic representation.

2.4 Profound and Coherent Mathematical Understanding

Ma Liping (1999) introduced the concept of Profound Understanding of Fundamental Mathematics, or PUFM. One of the defining characteristics of PUFM is longitudinal coherence, which means that every foundational concept must be developed through a consistent structure to prevent interference with subsequent concepts. Ma Liping specifically emphasizes that understanding fundamental mathematics is hierarchical and interdependent. In this context, inconsistencies in the number bases used in counting media do not merely constitute a visual or aesthetic issue. They represent a problem of conceptual coherence that may affect students' overall mathematical learning in the future.

2.5 Inconsistent Grouping and Base-Ten Understanding

Clements and Sarama (2004) explicitly state that using inconsistent grouping structures in early counting materials may interfere with children's construction of base-ten understanding. This finding directly relates to the inconsistencies identified in Gakko Kasho (2021), in which counting grids based on 5, 7, 9, and 10 are used interchangeably.

2.6 The Hierarchy of Mathematical Competencies

Kilpatrick, Swafford, and Findell (2001) systematically describe the hierarchy of fundamental mathematical competencies. Counting through a consistent number base constitutes the first foundation. This foundation subsequently supports the understanding of base ten, basic arithmetic operations involving addition, subtraction, multiplication, and division, basic addition facts, basic multiplication facts, place value and place-value-based operations, and advanced arithmetic operations such as exponentiation and root extraction. If the initial foundation becomes unstable because of inconsistent representations, the entire hierarchy built upon it may develop conceptual weaknesses.

2.7 Misconceptions Originating in the Initial Concept-Formation Stage

Ashlock (2010) demonstrates that most elementary school students' arithmetic misconceptions originate not during the operational stage but during the initial concept-formation stage. When visual representations introduced at the beginning of instruction are inconsistent with the intended mathematical structure, students may develop error patterns that become difficult to correct later, even after they have been taught the correct procedures.

2.8 The CRA Framework as a Solution for Representational Consistency

The Concrete–Representational–Abstract (CRA) framework is an evidence-based mathematics instructional approach that is structurally aligned with Bruner's theory of representation (1966). Within the CRA framework, students learn mathematical concepts progressively through three sequential and interconnected stages of representation: the concrete stage, which uses real objects and manipulatives; the representational or semi-concrete stage, which uses pictures, diagrams, or pictorial models; and the abstract stage, which uses numbers and mathematical symbols. Flores and Hinton (2022) emphasize that learning concepts through multiple structured representations promotes deep conceptual understanding and flexible mathematical thinking. The primary requirement for the effectiveness of the CRA framework is structural consistency across all three stages. Concrete objects, semi-concrete images, and abstract symbols must represent the same mathematical structure. In the context of base-ten counting instruction, this principle requires that: (a) the concrete objects represent a single group of ten; (b) the semi-concrete images reflect the same structure; and (c) the numerical symbols introduced to students remain connected to the two preceding representations. When one stage uses a different structure, such as base 5 or base 7, the connection between stages becomes disrupted, and the CRA framework loses its effectiveness (Flores & Hinton, 2022; Milton et al., 2023). Empirical research supports the effectiveness of the CRA framework in developing place-value understanding. Milton, Flores, Hinton, Dunn, and Darch (2023) found a functional relationship between the use of the CRA sequence and improvements in students' place-value understanding. These improvements included identifying the value represented by each digit, rounding three-digit numbers, and writing numbers in expanded form. Flores, Hinton, Shadoan, and Monroe (2024) also demonstrated that the integrated CRA sequence, known as CRA-I, effectively improved students' ability to perform subtraction involving regrouping and the decomposition of three-digit numbers. Both studies consistently used base-ten blocks as concrete instructional media. This selection reflects the principle of base consistency that forms the central argument of this article. In the context of first-grade elementary school counting instruction, a consistent base-ten application of the CRA framework can be implemented by developing concrete media in the form of a 1×10 counting-table card with buttons as number models. Semi-concrete media can take the form of a 1×10 counting-grid image that students complete by drawing circles with writing tools. Both forms of instructional media preserve the base-ten structure across all stages of representation. Consequently, transitions between the stages can occur smoothly, and the cognitive schemas developed by students can consistently lead to an understanding of the decimal system.

3. RESEARCH METHOD

This study employed content analysis using a qualitative descriptive approach. Content analysis is considered an appropriate research method for systematically and objectively examining the content of documents or written texts, including educational textbooks (Krippendorff, 2004). Through this method, the study examined how number-base structures were represented in counting-grid visuals and evaluated the consistency of these representations across the selected textbooks. The primary data sources consisted of two first-grade elementary school mathematics textbooks published by the Indonesian Ministry of Education, Culture, Research, and Technology. Both textbooks serve as official learning resources for the implementation of the Kurikulum Merdeka. The textbooks analyzed were as follows: Tim Gakko Kasho. (2021). *Belajar Bersama Temanmu: Matematika untuk Sekolah Dasar Kelas I.* Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia and Wulan, A. N. (2023). *Matematika untuk SD/MI Kelas I.* Ministry of Education, Culture, Research, and Technology of the Republic of Indonesia. These textbooks were selected because they are officially published instructional materials and are widely used in first-grade mathematics education under the current national curriculum. The analysis covered all pages containing counting-grid visuals in chapters or subchapters related to counting competencies and number recognition in Grade 1. Each individual presentation of a counting-grid visual served as a unit of analysis.

The analysis focused on four main aspects. First, it identified the number of cells contained within each frame or group of counting grids. Second, it examined the number base implicitly constructed through the visual structure of the media. Third, it evaluated the consistency of the number bases used across different pages and within the same page. Fourth, it assessed the extent to which each media format supported the intended construction of base-ten understanding. Through these analytical criteria, the study sought to determine whether the visual media provided a coherent and consistent representation of the decimal number system.

The data were analyzed through three main stages: inventory, categorization, and interpretation. The first stage involved creating an inventory of the data. At this stage, all counting-grid visuals presented in the two textbooks were identified, recorded, and documented. Relevant information, including page numbers, grid formats, and the number of cells in each visual, was systematically compiled. The second stage involved categorization. Each counting-grid visual was classified according to the number base represented by its structure. The categories included base 5, base 7, base 9, base 10, and combinations of more than one base. This classification made it possible to identify recurring patterns and variations in the number-base structures used throughout the textbooks. The final stage involved interpretation. The patterns identified during the categorization process were analyzed to determine their pedagogical and cognitive implications. The interpretation drew upon the theoretical frameworks discussed in the literature review, particularly theories concerning number-base construction, cognitive schema development, representational consistency, and the Concrete–Representational–Abstract framework. This analytical process enabled the study to explain not only the forms of inconsistency found in the textbooks but also their potential effects on students' understanding of counting, place value, and the base-ten number system.

4. RESULTS AND DISCUSSION

3.1 Findings from Tim Gakko Kasho's Textbook (2021)

The analysis of Tim Gakko Kasho's textbook (2021) revealed the use of counting-grid visuals based on several different number bases within the same textbook and, in some cases, even on the same page. More specifically, the textbook employs counting grids based on 5, 7, 9, and 10. All four bases appear within the same chapter, namely the chapter addressing number recognition and early counting instruction for first-grade elementary school students. On several pages, two different number bases are presented simultaneously. For example, one side of a page may contain a five-cell counting grid, while the other side displays a ten-cell grid. Other pages place a seven-cell grid next to a nine-cell grid. Presenting two different number bases on the same page may create cognitive confusion among first-grade students who are still developing their initial numerical schemas.

3.2 Findings from Wulan's Textbook (2023)

The analysis of Wulan's textbook (2023) revealed a different pattern. This textbook consistently uses a single counting-grid format, namely the 2 × 5 format, which consists of two rows with five cells in each row. This format appears throughout all pages containing counting materials. Although this textbook demonstrates greater consistency than Gakko Kasho (2021) in terms of format uniformity, the 2 × 5 structure inherently establishes groupings of five before forming a complete group of ten. The two rows, each containing five cells, visually emphasize two groups of five rather than one unified group of ten. This structure may contribute to the development of an intermediate cognitive schema based on five before students construct the intended base-ten schema.

3.3 Summary of the Findings

Summarizes the findings obtained from the content analysis of the two textbooks.

Table 1. Summary of Number-Base Inconsistencies in Counting-Grid Media

No	Textbook	Page or Section	Number Base Used	Notes
1	Tim Gakko Kasho (2021)	Chapter 1, pp. 12–15	Bases 5 and 10	Two different bases appear within the same chapter
2	Tim Gakko Kasho (2021)	pp. 16–18	Base 7	A seven-cell counting grid is used without pedagogical justification
3	Tim Gakko Kasho (2021)	pp. 19–22	Base 9	Base 9 appears without a clear connection to base 10
4	Tim Gakko Kasho (2021)	Several pages	Two different bases	Two number bases appear simultaneously on the same page
5	Wulan (2023)	Entire counting chapter	Base 5 through a 2 × 5 structure	The 2 × 5 ten-frame format establishes base-5 grouping before base 10

3.4 Comparative Analysis of Media Formats

Based on the theoretical framework proposed by Van de Walle et al. (2013), counting-grid formats differ substantially in terms of the number-base structures they cognitively construct. Table 2 compares the media formats and evaluates their suitability for developing base-ten understanding.

Table 2. Comparison of Counting-Grid Formats and Their Suitability for Base-Ten Construction

Media Format	Visual Structure	Constructed Number Base	Suitability for the Intended Purpose
1 × 10 grid, one row	Linear structure with ten consecutive cells	Direct base-ten structure	✓ Highly suitable
2 × 5 grid, two rows	Two groups of five	Base 5 followed by base 10	△ Partially suitable
Base-7 or base-9 grid	Nonstandard grouping	Irrelevant number base	✗ Unsuitable
Mixed-base grid, with more than one base per chapter	Inconsistent structure	Conflicting schemas	✗ Counterproductive

As shown in **Table 2**, the 1 × 10 grid, consisting of a single row of ten cells, is the most consistent format and provides the most direct representation of the decimal structure. This format helps students visualize and intuitively understand base-ten number relationships without first passing through an intermediate grouping structure.

3.5. Pedagogical Effects of Number-Base Inconsistency

Based on the theoretical frameworks discussed earlier, inconsistencies in the number bases represented in counting-grid visuals may produce at least four categories of pedagogical consequences. Table 3 summarizes these effects and their theoretical foundations.

Table 3. Pedagogical Effects of Number-Base Inconsistency in Counting Media

Type of Effect	Description	Theoretical Foundation
Number-base misconceptions	Students fail to construct a complete base-ten concept and develop an incomplete mental representation	Ashlock (2010); Clements and Sarama (2004)
Cognitive schema conflict	Base-5 and base-7 schemas compete with the base-ten schema in students' working memory	Piaget (1965); Bruner (1966)
Longitudinal learning difficulties	Students experience difficulties in understanding basic arithmetic operations, addition and multiplication facts, place value, and place-value-based operations in subsequent grades	Kilpatrick et al. (2001); Ma Liping (1999)
Disruption of representational transitions	The transition from iconic to symbolic representation becomes obstructed because the representations lack a consistent structural connection	Bruner (1966); Ding and Li (2014)

3.6 Discussion

Inconsistency as a Problem of Iconic Representation

The findings can be understood as evidence of inconsistency in the iconic representations used in first-grade elementary school mathematics textbooks. Within Bruner's theoretical framework (1966), iconic representation functions as a bridge between concrete experience and abstract understanding. When this bridge lacks consistency and uses changing structures, the transition toward the symbolic representation of base ten, namely the decimal number system, becomes disrupted. Charalambous, Delaney, Hsu, and Mesa (2010) found that inconsistencies in textbook representations constitute one of the factors contributing to weak conceptual understanding among students across different countries. More specifically, Ding and Li (2014) found that textbooks that employ multiple visual models without a clear instructional direction may slow the formation of abstract concepts among students in the early grades. In the Indonesian context, these findings have serious implications. Textbooks published by the Indonesian Ministry of Education, Culture, Research, and Technology have extensive national coverage and are used in elementary schools throughout the country. Inconsistencies in the representations presented in these official textbooks may affect the development of mathematical cognitive schemas among millions of first-grade students. These early cognitive effects may subsequently influence the development of their mathematical competencies in higher grades. This concern becomes increasingly significant because counting occupies a strategic position within the hierarchy of mathematical competencies. Kilpatrick et al. (2001) emphasize that when the initial foundation, namely counting with a consistent number base, is unstable, the entire hierarchy built upon it may develop conceptual weaknesses. This hierarchy includes basic arithmetic operations, basic addition and subtraction facts, place value, and place-value-based operations. As demonstrated by Ashlock (2010), misconceptions developed during the initial learning stage are often difficult to correct, even after students receive accurate instruction in higher grades.

3.7. Recommended Solution

Base-Ten Concrete and Semi-Concrete Media

Drawing on the CRA framework and Bruner's theory of representation, the inconsistencies identified in the textbooks can be addressed through the development and use of two types of instructional media that consistently employ a base-ten structure. These media may supplement or replace inconsistent counting-grid visuals in existing textbooks. They can also ensure that all stages of representation, from concrete to symbolic, construct the same base-ten schema. The first recommended medium is a concrete or enactive instructional tool in the form of a 1 × 10 counting-table card equipped with buttons as number models. The card consists of one row containing ten equally sized square cells. This structure directly represents a single group of ten. Students place the buttons into the cells one at a time according to the number being studied. The use of buttons as concrete manipulatives is not new in elementary mathematics education research. Studies conducted in Indonesia have shown that concrete objects such as beads, buttons, and similar materials can increase first-grade students' enthusiasm, active participation, and conceptual understanding (Hasanah & Nurjanah, 2021; Aulia et al., 2024). The novelty of the proposed medium lies in the explicit placement of the buttons on a 1 × 10 structured card. Consequently, a concrete medium that has already demonstrated instructional effectiveness can also ensure consistent base-ten representation. The second recommended medium is a semi-concrete or iconic instructional tool in the form of a 1 × 10 counting-grid worksheet. Students can complete the grid using writing tools in several ways, such as placing crosses, drawing circles, or coloring the cells. Drawing circles is preferable because the circular marks correspond visually to the buttons used during the enactive stage.

This worksheet serves as an iconic bridge between students' concrete experiences with buttons and their abstract understanding of numbers. The 1 × 10 structure of the worksheet is identical to the structure of the concrete counting-table card. Therefore, the structural connection between the concrete and iconic stages remains fully intact. Flores and Hinton (2022) emphasize that this form of structural consistency across representational stages is central to the effectiveness of the CRA framework in developing deep mathematical understanding. From a pedagogical perspective, using both media within a single instructional sequence creates a coherent learning experience. First, students count real objects, namely buttons, while placing them on the 1 × 10 counting-table card during the enactive or concrete stage. Second, students count and mark the cells on the 1 × 10 counting-grid worksheet during the iconic or semi-concrete stage. Third, students write the numerical symbol that represents the quantity they have counted during the symbolic or abstract stage. Across these three stages, the base-ten structure remains consistent. No transition to base 5, base 7, or base 9 occurs during the instructional process. As a result, students develop a pure and coherent base-ten cognitive schema from the beginning. This schema can provide a strong foundation for learning place value and arithmetic operations in subsequent grades (Kilpatrick et al., 2001; Mix et al., 2022).

5. CONCLUSION

Based on the content analysis of two first-grade elementary school mathematics textbooks published by the Indonesian Ministry of Education, Culture, Research, and Technology, this study concludes that significant inconsistencies exist in the number bases represented through counting-grid visuals. Tim Gakko Kasho (2021) uses grids based on 5, 7, 9, and 10 interchangeably within the same chapter. Several pages even present two different number bases simultaneously. In contrast, Wulan (2023) consistently uses a 2 × 5 grid format. Although this format provides greater visual uniformity, it emphasizes two groups of five before forming one complete group of ten. Therefore, it may encourage students to develop an intermediate base-5 schema before establishing a base-ten schema. These inconsistencies should not be viewed merely as visual or aesthetic issues. They represent problems of conceptual coherence that may contribute to misconceptions about number bases, conflicts between cognitive schemas, disruptions in the transition from iconic to symbolic representation, and long-term difficulties in understanding place value and place-value-based arithmetic operations. This interpretation is supported by the theoretical perspectives of Piaget, Bruner, Ma Liping, Van de Walle, Clements and Sarama, Kilpatrick et al., and Ashlock, who emphasize the importance of stable, coherent, and interconnected representations in early mathematics learning. The study therefore concludes that counting instruction in Grade 1 should consistently employ a base-ten structure across concrete, iconic, and symbolic representations. A 1 × 10 counting-table card with buttons can function as concrete or enactive media, while a 1 × 10 counting-grid worksheet can serve as semi-concrete or iconic media. When implemented within the Concrete–Representational–Abstract framework, these media can maintain structural consistency throughout the learning process and support the development of a coherent base-ten cognitive schema.

RECOMMENDATIONS

The Indonesian Ministry of Education, Culture, Research, and Technology should revise the counting-grid visuals used in first-grade mathematics textbooks. The 1 × 10 grid format should become the primary representation in all counting materials because it introduces base ten directly and explicitly. The use of nonstandard grids based on 7 or 9 and the simultaneous presentation of different number bases should be avoided during the initial stage of counting instruction. A national standard for the use of ten-frames in first-grade mathematics textbooks should also be established. The standard should prioritize the 1 × 10 format rather than the 2 × 5 format. Although the 2 × 5 format can support number decomposition and subitizing, the 1 × 10 format more directly represents one complete group of ten and is therefore more appropriate for the initial construction of base-ten understanding. Teachers and instructional-material developers should provide a 1 × 10 counting-table card with buttons during the concrete stage and a structurally identical 1 × 10 counting-grid worksheet during the iconic stage. Students should first count and place physical objects on the card, then represent the same quantities by drawing circles, placing marks, or coloring cells on the worksheet, and finally write the corresponding numerical symbols. This sequence can preserve the connection between concrete experience, visual representation, and abstract notation. First-grade teachers should receive training on the CRA framework, the importance of number-base consistency, and the effective use of base-ten instructional media. The training should also prepare teachers to identify inconsistent representations in textbooks and make appropriate instructional adjustments when the available materials do not adequately support base-ten understanding. Further empirical studies should examine the effectiveness of 1 × 10 counting-table cards and 1 × 10 counting-grid worksheets in developing students' counting skills, base-ten schemas, place-value understanding, and arithmetic abilities. Researchers may employ classroom action research, quasi-experimental designs, or longitudinal studies to determine the immediate and long-term effects of these media. The proposed media may also incorporate culturally relevant and locally available materials. In Papua and other regions, teachers may replace buttons with objects such as areca nuts, shells, seeds, or other natural materials. However, these objects should remain arranged within a 1 × 10 structure so that contextual learning does not compromise base-ten consistency. Other number systems, such as base two and base five, may be introduced after students have developed a stable understanding of base ten. At this later stage, students can compare different number systems, examine their functions and relationships, and explore number-base practices found in different cultural or regional contexts. This sequence can broaden students' mathematical understanding without disrupting the initial development of a coherent base-ten schema.

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